TABLE I

CALCULATION OF CPW TRANSMISSION LINE[a]

	Infinite Substrate Approximation	Thickness= (b_1-a_1)		Thickness= $3(b_1-a_1)$	
		Finite Substrate Conformal Mapping	Finite Element Field Map	Finite Substrate Conformal Mapping	Finite Element Field Map
Relative Phase Vel. v_{ph}	0.43	0.48	0.50	0.43	0.44
Characteristic Impedance, Z_0	51.4	57.7	58.5	52.4	53.0
Percent of Energy in Substrate	91.1	79.0	80.0	89.0	90.0

[a] Characteristics for $\epsilon_r=10$. $(a_1/b_1=0.5.)$

below the dielectric boundary as the slot-width to substrate-thickness ratio decreases.

MEASUREMENTS

A number of CPW lines were fabricated on alumina substrates. The line impedances were measured on an Hp 1815B time-domain reflectometer with a 28-ps rise-time pulse. These results are the solid data points shown on Fig. 3 for the lines designed for 50 Ω from Wen's evaluation $(a_1/b_1=0.5)$. The other data points were obtained by McDade [4]. As can be seen, good qualitative results were obtained. It was quite necessary, however, in making the measurements to assure that the substrates were suitably suspended in air, since any metal ground planes affected the impedance measurements. These results were substantiated by McDade [4] who fabricated similar circuits both with and without ground planes. Those circuits with ground planes showed a marked preference for microstrip modes and impedance levels as opposed to CPW.

CONCLUSIONS

The effects of finite-substrate thickness on the characteristics of CPW are important in designing circuits where close control of the transmission-line impedance is needed. It is shown that deviation from the results of an infinite dielectric can approach 10–15 percent for substrates whose thicknesses are less than two times the gap width. These calculations were all made using a static TEM approximation and further corrections would be present if dispersion were considered. Finally, the circuits fabricated showed a pronounced tendency to be affected by external metallic walls, which would be a highly undesirable effect if use of these transmission lines as microwave integrated circuits is anticipated.

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REFERENCES

[1] C. P. Wen, "Coplanar waveguide: A surface strip transmission line suitable for nonreciprocal gyromagnetic device applications," *IEEE Trans. Microwave Theory Tech.* (*1969 Symposium Issue*), vol. MTT-17, pp. 1087–1090, Dec. 1969.
[2] ——, "Coplanar-waveguide directional couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 318–322, June 1970.
[3] F. J. Rosenbaum *et al.*, Final Rep. AFAL-TR-71-313; also AF Contract 33615-70-C-1456, 1971, p. 88.
[4] J. McDade, "MIC transmission line techniques," General Electric Co., Utica, N. Y., to be reported under AF Contract 33615-72-C-1949.
[5] M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions* (Applied Mathematics Series 55). Washington, D. C.: NBS, 1964, p. 589.
[6] H. A. Wheeler, "Transmission-line properties of parallel strips separated by a dielectric sheet," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 172–185, Mar. 1965.
[7] Z. J. Csendes and P. Silvester, "Numerical solution of dielectric loaded waveguides: I–Finite-element analysis," *IEEE Trans. Microwave Theory Tech.* (*1970 Symposium Issue*), vol. MTT-18, pp. 1124–1131, Dec. 1970.

Letters

Analysis of Microwave Circuit for Characterization of Negative-Conductance Devices by Transients

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Abstract—**The assumptions required for the transient method of measuring Gunn-diode conductances are shown to be valid if either the diode susceptance or the characteristic admittance of the resonator transmission line are larger than the modulus of the negative conductance of the device.**

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A method for measuring the negative conductance of Gunn diodes from the envelope of the transient amplitude of oscillations was reported recently [1], [2]. This new method is useful since the measurement of the diode conductance has been possible so far, in those cases where either devices can be stabilized by heavy loading or where a reasonably correct load impedance of oscillating diodes can be measured separately, when only the steady-state conductance is obtained.

The conductance measurement by transient recording depends on the assumptions that the ratio of the voltage across the diode

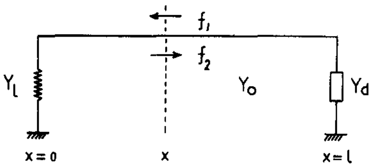

Fig. 1. Representation of microwave cavity by a one-dimensional line of length l.

(v_d) to that across the load (v_l) is independent of the value of diode conductance G_d and that the oscillating frequency is constant during the transient state. Here we derive the conditions for which these assumptions are valid.

The cavity is represented in Fig. 1 as a lossless one-dimensional distributed line of length l. The voltage and current distributions as a solution of the wave equation are given in (1) by the linear combination of two arbitrary funcrions $f_1(x, t)$ and $f_2(x, t)$ that propagate on the line in opposite directions:

$$\begin{aligned} v(x, t) &= f_1(x, t) + f_2(x, t) \\ i(x, t) &= -\ Y_0[f_1(x, t) - f_2(x, t)]. \end{aligned} \tag{1}$$

The boundary condition at $x=0$ gives

$$\begin{aligned} v(x, t) &= f_1(x, t) + \Gamma_l \cdot f_1(x, t - \tau_1) \\ i(x, t) &= -Y_0[f_1(x, t) - \Gamma_l \cdot f_1(x, t - \tau_1)] \end{aligned} \tag{2}$$

where

- Y_0 characteristic admittance of the line;
- Y_l admittance of the load;
- Γ_l $(Y_0 - Y_l)/(Y_0 + Y_l)$, the reflection coefficient at the load end;
- τ_1 $=2\,x/u$;
- u velocity of wave propagation on the line.

Γ_l is a real number, because Y_l is assumed to be real for simplicity. If we consider the other boundary condition at $x=l$ and substitute l into x, we obtain (3):

$$(Y_0 - Y_d)\Gamma_l \cdot f_1(l, t - \tau) = (Y_0 + Y_d) \cdot f_1(l, t) \tag{3}$$

where

- τ $=2\,l/u$;
- Y_d $=G_d + jB_d$;
- G_d conductance of the diode;
- B_d susceptance of the diode.

Instead of considering a suitable initial condition on the line, we consider a time-growing sinusoidal oscillation at $x=l$ as follows:

$$\begin{aligned} f_1(l, t) &= \Psi \epsilon^{\lambda t} \\ &= \Psi \epsilon^{\alpha t}(\cos \omega t + j \sin \omega t). \end{aligned} \tag{4}$$

Substituting (4) into (3) and considering $Y_d = G_d + jB_d$, we obtain

$$\begin{aligned} \epsilon^{\alpha\tau} &= \sqrt{\xi^2 + \eta^2} \\ \sin \omega\tau &= \frac{\eta}{\sqrt{\xi^2 + \eta^2}} \end{aligned} \tag{5}$$

where

$$\begin{aligned} \xi &= \Gamma_l \cdot \frac{Y_0^2 - G_d^2 - B_d^2}{(Y_0 + G_d)^2 + B_d^2} \\ \eta &= -\ \Gamma_l \cdot \frac{2Y_0B_d}{(Y_0 + G_d)^2 + B_d^2}. \end{aligned} \tag{6}$$

The angular frequency and the growth rate of the wave are shown in (5). Since the diode conductance G_d and the diode susceptance B_d are generally dependent on the amplitude of ac voltage, v_d across the diode, the angular frequency and the growth rate are time-dependent values as a result of the behaviour of ξ and η.

The ratio of the voltage across the diode to that across the load is evaluated from (2)–(4) as follows:

$$\frac{v_d(t)}{v_l(t)} = \frac{\epsilon^{\lambda\tau/2} + \Gamma_l \epsilon^{-\lambda\tau/2}}{1 + \Gamma_l}. \tag{7}$$

Using (5) and (6), we evaluate the ratio of the squared absolute value of this, because we do not need to consider the phase relation.

$$\left(\frac{|v_d(t)|}{|v_l(t)|}\right)^2 = \frac{|\Gamma_l|}{(1 + \Gamma_l)^2} \frac{4Y_0^2}{\sqrt{(Y_0^2 - G_d^2 - B_d^2)^2 + 4Y_0^2B_d^2}}. \tag{8}$$

The squared voltage ratio in (8) is a function of the diode conductance G_d and the diode susceptance B_d once the values of Γ_l and Y_0 are fixed. We can only assume the existence of a constant voltage ratio during the transient oscillation when either $|G_d| \ll B_d$ or $|G_d| \ll Y_0$. Then one has

$$\left(\frac{|v_d(t)|}{|v_l(t)|}\right)^2 = \frac{|\Gamma_l|}{(1 + \Gamma_l)^2} \cdot \frac{4Y_0^2}{Y_0^2 + B_d^2}. \tag{9}$$

This shows that the voltage ratio becomes independent of the diode conductance.

Under the same conditions ($|G_d| \ll B_d$ or $|G_d| \ll Y_0$), the angular frequency of oscillation as given by (5) becomes independent of G_d.

Therefore, if the diode susceptance is independent of the ac amplitude or if an equivalent diode susceptance becomes constant for given experimental conditions, the determination of the diode conductance from the transient envelope of oscillations becomes possible.

Under the conditions of $|G_d| \ll B_d$ or $|G_d| \ll Y_0$, the negative conductance of the diode delivers only a small amount of power to the cavity, and the resulting slow transient oscillation then behaves like a steady-state case.

References

[1] M. Kawashima and H. Hartnagel, "New measurement method of Gunn-diode impedance," *Electron. Lett.*, vol. 8, no. 12, pp. 305–306, 1972.
[2] H. Hartnagel and M. Kawashima, "Negative TEO-diode conductance by transient measurement and computer simulation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 468–477, July 1973.

Comments on "Variational Formulation of the Dirichlet Boundary Condition"

SHIMON COEN

In the above paper,[1] (21) does not satisfy Neumann's boundary condition

$$\left.\frac{\partial \phi}{\partial n}\right|_{y=1} = 0$$

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[1] T. G. Hazel and A. Wexler, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 385–390, June 1972.